

$f(x)$  is a polynomial function

SCORE: \_\_\_\_ / 6 PTS

with derivative  $f'(x) = (x+2)(x-7)^2(x+6)^3$  and second derivative  $f''(x) = 6(x^2-10)(x-7)(x+6)^2$ .

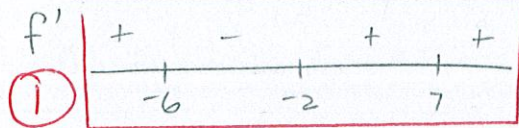
- [a] Find the critical numbers of  $f$ . Justify your answer very briefly.

$\frac{1}{2}$  DOMAIN OF  $f = (-\infty, \infty)$  SINCE  $f$  IS A POLY

$\frac{1}{2}$   $f' = 0$  @  $x = -2, 7, -6$  ①

- [b] Run the First Derivative Test for Local Extrema on each critical number, and state what it tells you about that critical number.

Justify your answer very briefly. Do NOT use the Second Derivative Test.



$f'$  CHANGES FROM + TO -, SO LOCAL MAX @ -6,  $\frac{1}{2}$

- + LOCAL MIN @ -2,  $\frac{1}{2}$

DOES NOT CHANGE SIGN, SO NO LOCAL EXTREMA @ 7,  $\frac{1}{2}$

- [c] Run the Second Derivative Test for Local Extrema on each critical number, and state what it tells you about that critical number.

Justify your answer very briefly. Do NOT use the First Derivative Test.

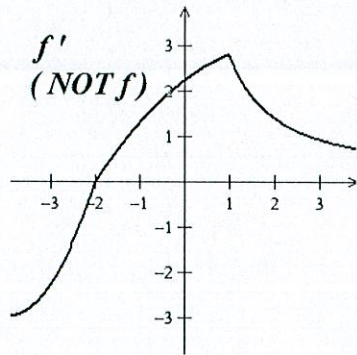
$f''(-6) = f''(7) = 0$ , SO 2<sup>ND</sup> DERIV TEST SAYS NOTHING ABOUT -6, 7, ①

$f''(-2) > 0$ , SO LOCAL MIN @ -2,  $\frac{1}{2}$

$f(x)$  is a continuous function whose derivative  $f'(x)$  is shown on the right.

SCORE: \_\_\_\_\_ / 5 PTS

The following questions are about the function  $f$ , **NOT THE FUNCTION  $f'$** .



[a] Write "I UNDERSTAND" if you understand that the following questions are about the continuous function  $f$ , **NOT THE FUNCTION  $f'$** .

[b] Find all intervals over which  $f$  is concave down.

**Justify your answer very briefly.**

$\frac{1}{2}$   $f'$  DECR ON  $(1, \infty)$  ①

[c] Find all intervals over which  $f$  is decreasing.

**Justify your answer very briefly.**

$\frac{1}{2}$   $f' < 0$  ON  $(-\infty, -2)$  ①

[d] Find the  $x$ -coordinates of all local extrema of  $f$  and identify whether they are local maxima or minima.

**Justify your answer very briefly.**

$f'$  CHANGES FROM - TO + ①  
SO LOCAL MIN @  $x = -2$  ①



Graph  $f(x) = 25x^{\frac{3}{5}}(x+8)$  using the process shown in lecture and in the website handout.

SCORE: \_\_\_\_ / 19 PTS

Complete the table at the bottom of the page, after showing relevant work (you do NOT need to show work for entries marked ★). You will NOT receive credit for the entries in the table if the relevant work is missing.

y-INT:  $f(0) = 25(0)(8) = 0$

x-INT:  $25x^{\frac{3}{5}}(x+8) = 0 \rightarrow x=0$  OR  $x=-8$

$\lim_{x \rightarrow \infty} 25x^{\frac{3}{5}}(x+8) = \infty$  ( $\infty \cdot \infty$ )  $\left(\frac{1}{2}\right)$  POINT

$\lim_{x \rightarrow -\infty} 25x^{\frac{3}{5}}(x+8) = \infty$  ( $-\infty \cdot -\infty$ ) EACH ITEM UNLESS OTHERWISE LABELLED

$\lim_{x \rightarrow 0^+} f'(x) = \lim_{x \rightarrow 0^+} 40x^{-\frac{2}{5}}(x+3) = \lim_{x \rightarrow 0^+} \frac{40(x+3)}{x^{\frac{2}{5}}} = \infty$  ( $\frac{120}{0^+}$ )

$\lim_{x \rightarrow 0^-} \frac{40(x+3)}{x^{\frac{2}{5}}} = \infty$  ( $\frac{120}{0^+}$ )

$f(x) = 25x^{\frac{8}{5}} + 200x^{\frac{3}{5}}$

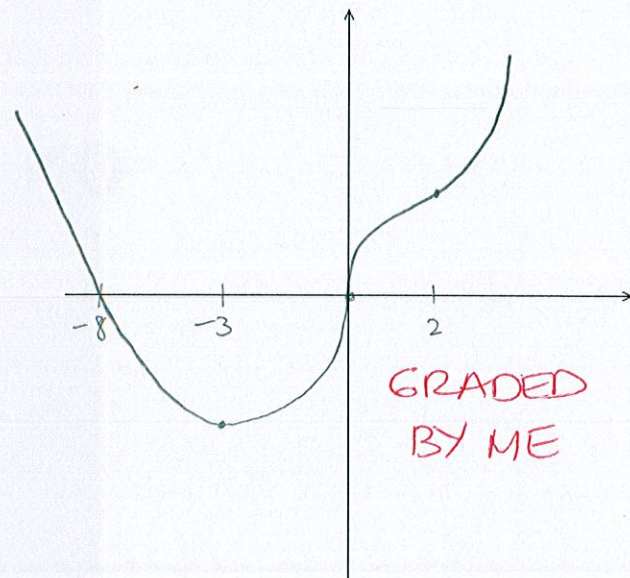
$f'(x) = 40x^{\frac{3}{5}} + 120x^{-\frac{2}{5}} = 40x^{-\frac{2}{5}}(x+3)$

$f''(x) = 24x^{-\frac{7}{5}} - 48x^{-\frac{7}{5}} = 24x^{-\frac{7}{5}}(x-2)$

$f'$  DNE @  $x=0$        $f''$  DNE @  $x=0$

$f' = 0$  @  $x = -3$        $f'' = 0$  @  $x = 2$

$f'$	-	MIN	+		+		+	②
$f''$	+		+	IP	-	IP	+	②
	-3		0		2			
	$(-3, -125 \cdot 3^{\frac{3}{5}})$		$(0, 0)$		$(2, 250 \cdot 2^{\frac{3}{5}})$			
	H.T.L.		V.T.L.					



★ Domain	★ Discontinuities	Intercepts (specify x- or y-)	One sided limits at each discontinuity (write using proper limit notation)	
$(-\infty, \infty)$	NONE	x-INT: -8, 0 y-INT: 0	N/A	
Equations of Horizontal Asymptotes	Intervals of Increase	Intervals of Decrease	Intervals of Upward Concavity	Intervals of Downward Concavity
NONE	$(-3, \infty)$	$(-\infty, -3)$	$(-\infty, 0) (2, \infty)$	$(0, 2)$
Vertical Tangent Lines (x-coordinates)	Horizontal Tangent Lines (x-coordinates)	Local Maxima (x-coordinates)	Local Minima (x-coordinates)	Inflection Points (x-coordinates)
$x=0$	$x=-3$	NONE	$x=-3$	$x=2$